Hydrodynamics Schemes: Spoilt for Choice

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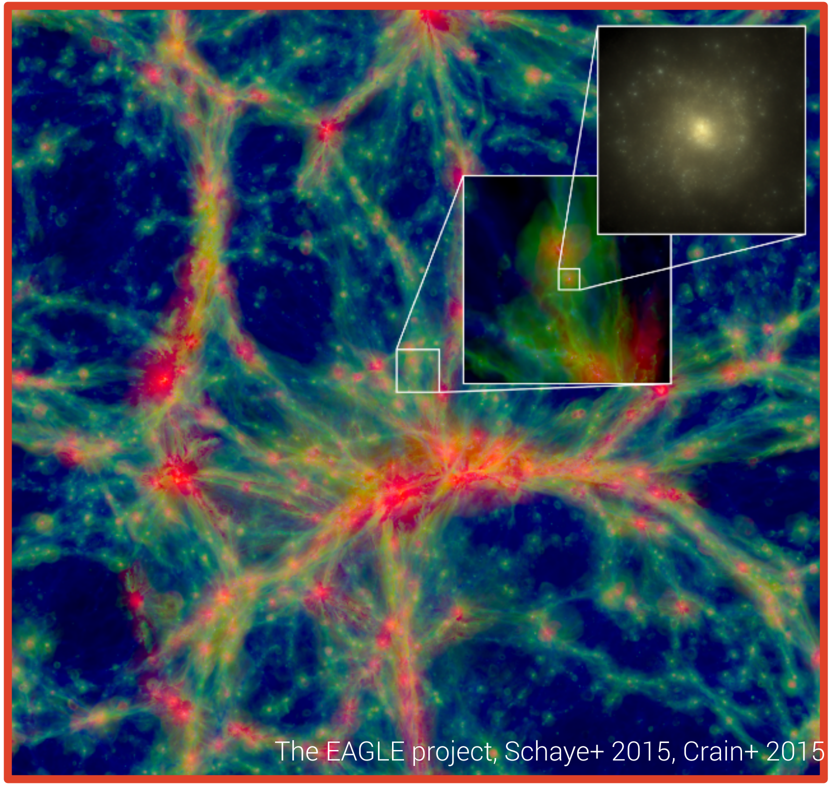
Overview

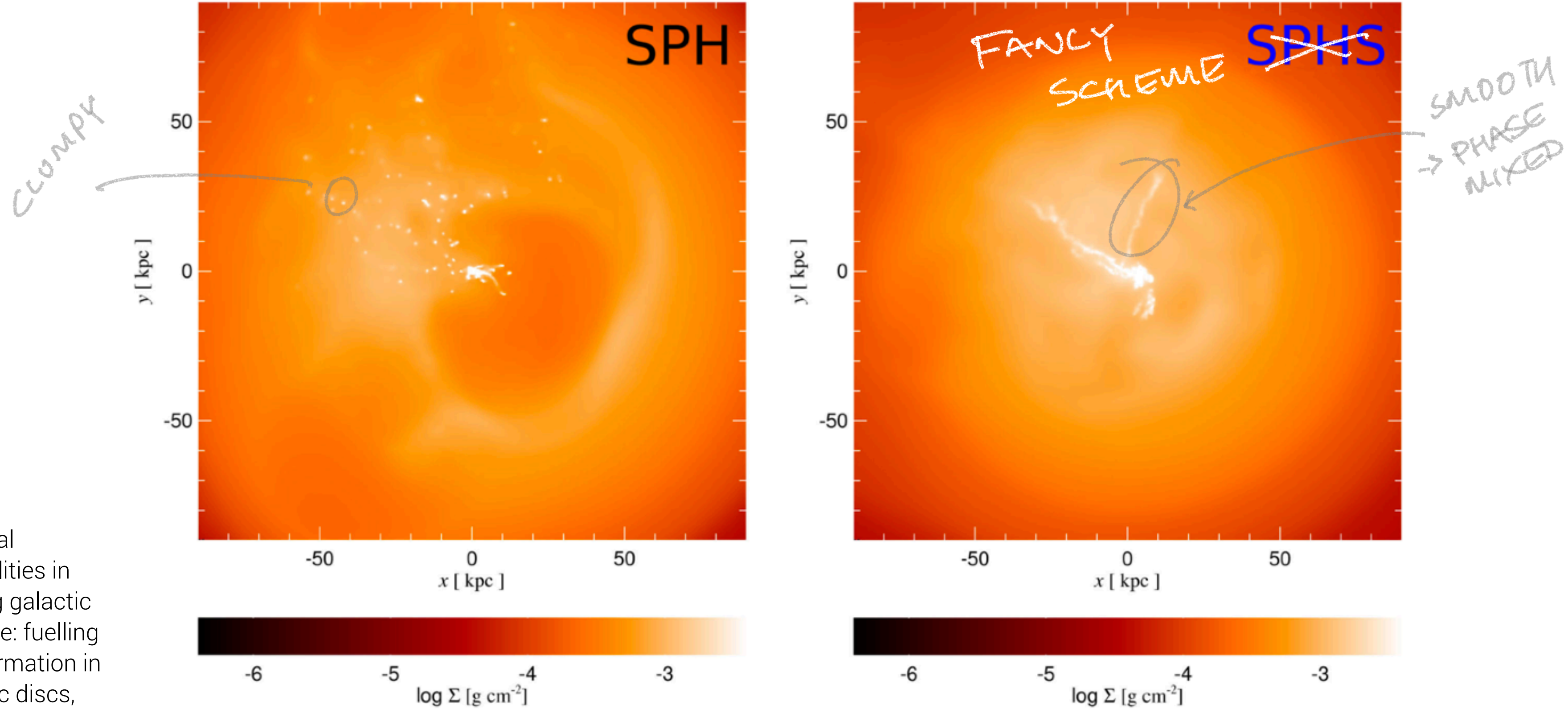
- Introduction to Cosmological Simulations
- What's the problem with old-fashioned SPH?
- Implementing a new hydrodynamics scheme
- Re-phrasing the problem: going down the abstraction hierarchy

Cosmological Simulations

- Need to solve gravity and hydrodynamics (along with a sub-grid model)
- Speed prized over accuracy: bigger box-sizes, higher resolution for sub-grid physics, etc.



What's the Problem? (Physics)

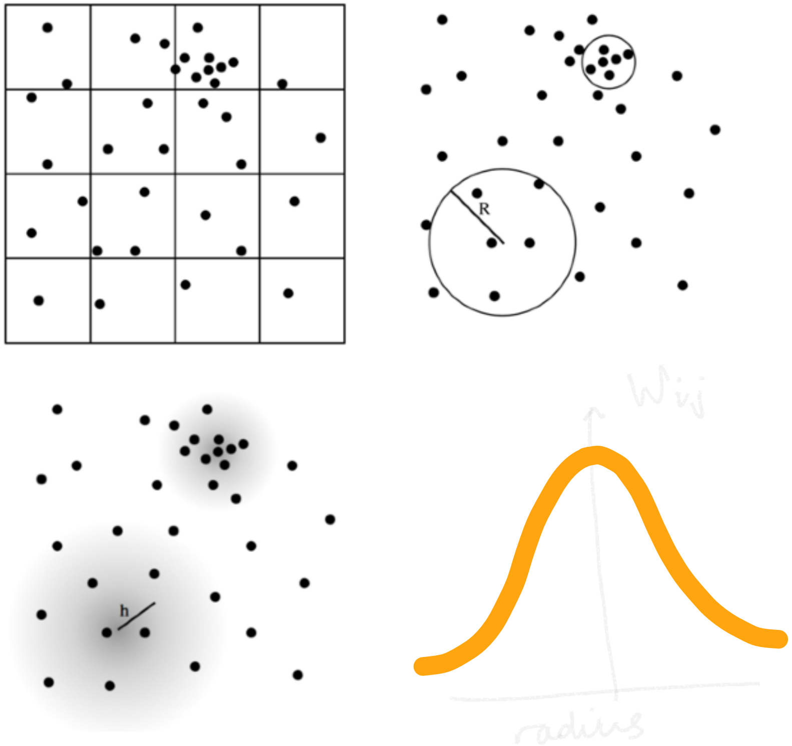


Thermal instabilities in cooling galactic coronae: fuelling star formation in galactic discs, Hobbs+, 2013

Density from Particles

- Say I am given a distribution of particles of mass *m*. What is the density?
- Can use a kernel-weighted average.

$$\rho_i = \sum_j m_j W_{ij}$$

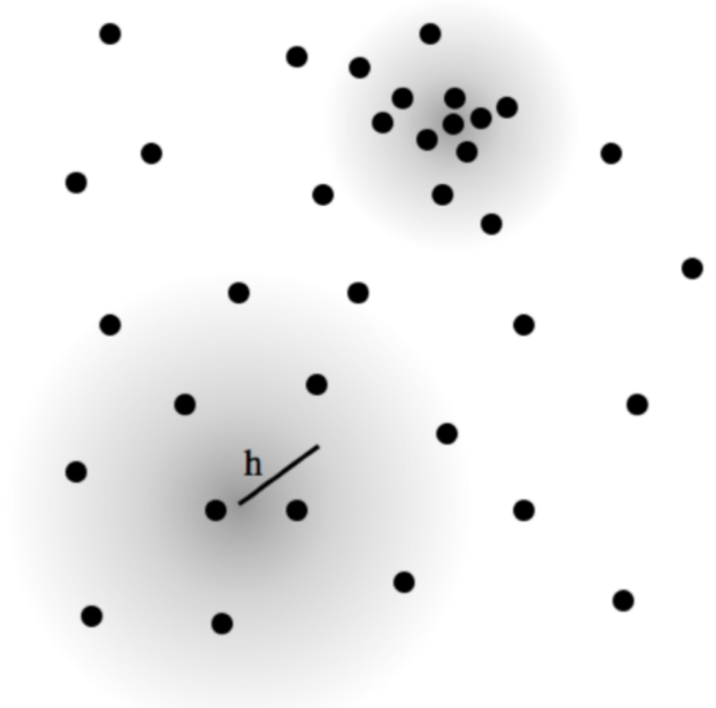


Smoothed Particle Hydrodynamics and Magnetohydrodynamics, Price, 2012

Smoothed Particle Hydrodynamics

- General method used both in astrophysics and industry
- Represent the fluid as particles (Lagrangian)

$$L(q, \dot{q}) = \frac{1}{2}\sum_{i=1}^{N} m_i \dot{r}_i^2 - \sum_{i=1}^{N} m_i u_i,$$



Smoothed Particle Hydrodynamics and Magnetohydrodynamics, Price, 2012

A Quick Derivation

- Add in a little 1st law...

INTERNAL ENERGY

PRESSURE

$$\left.\frac{\partial u_i}{\partial q_i}\right|_A = -\frac{P_i}{m_i}\frac{\partial \Delta V_i}{\partial q_i}$$

VOLUME ELEMENT

x/y

GENERALIZED CO-ORDINATE

PARTICLE MASS

- A sprinkle of constraint equation...

3

NUMBER OF NEIGHBOURS

$$\phi_i(\mathbf{q}) = \kappa h_i^{n_d}\frac{1}{\Delta\tilde{V}} - N_{ngb} = 0$$

4π/3

A general class of Lagrangian smoothed particle hydrodynamics methods and implications for fluid mixing problems, Hopkins, 2013

A General Equation of Motion

- Follow the equations through (Lagrange multipliers)

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_{j=1}^{N} x_i x_j \left[\frac{f_{ij} P_i}{y_i^2} \nabla_i W_{ij}(h_i) + \frac{f_{ji} P_j}{y_j^2} \nabla_i W_{ji}(h_j) \right]$$

- With a correction for non-constant smoothing lengths

$$f_{ij} \equiv 1 - \frac{\tilde{x}_j}{x_j} \left(\frac{h_i}{n_d \tilde{y}_i} \frac{\partial y_i}{\partial h_i} \right) \left(1 + \frac{h_i}{n_d \tilde{y}_i} \frac{\partial \tilde{y}_i}{\partial h_i} \right)^{-1}$$

Getting an "Actual Scheme"

- Now need to make a *choice* of volume element.

$$P_{\mathrm{eos}} = (\gamma - 1)\rho u$$

← EQUATION OF STATE

Density-Energy

y

$$\rho_i = \sum_j \underbrace{m_j}_{x} W_{ij}$$

$$\Delta V = \frac{m}{\rho}$$

Pressure-Energy

y

$$\bar{P}_i = \sum_j \underbrace{m_j u_j (\gamma - 1)}_{x} W_{ij}$$

$$\Delta V = \frac{(\gamma - 1)mu}{\bar{P}}$$

Getting an “Actual Scheme”

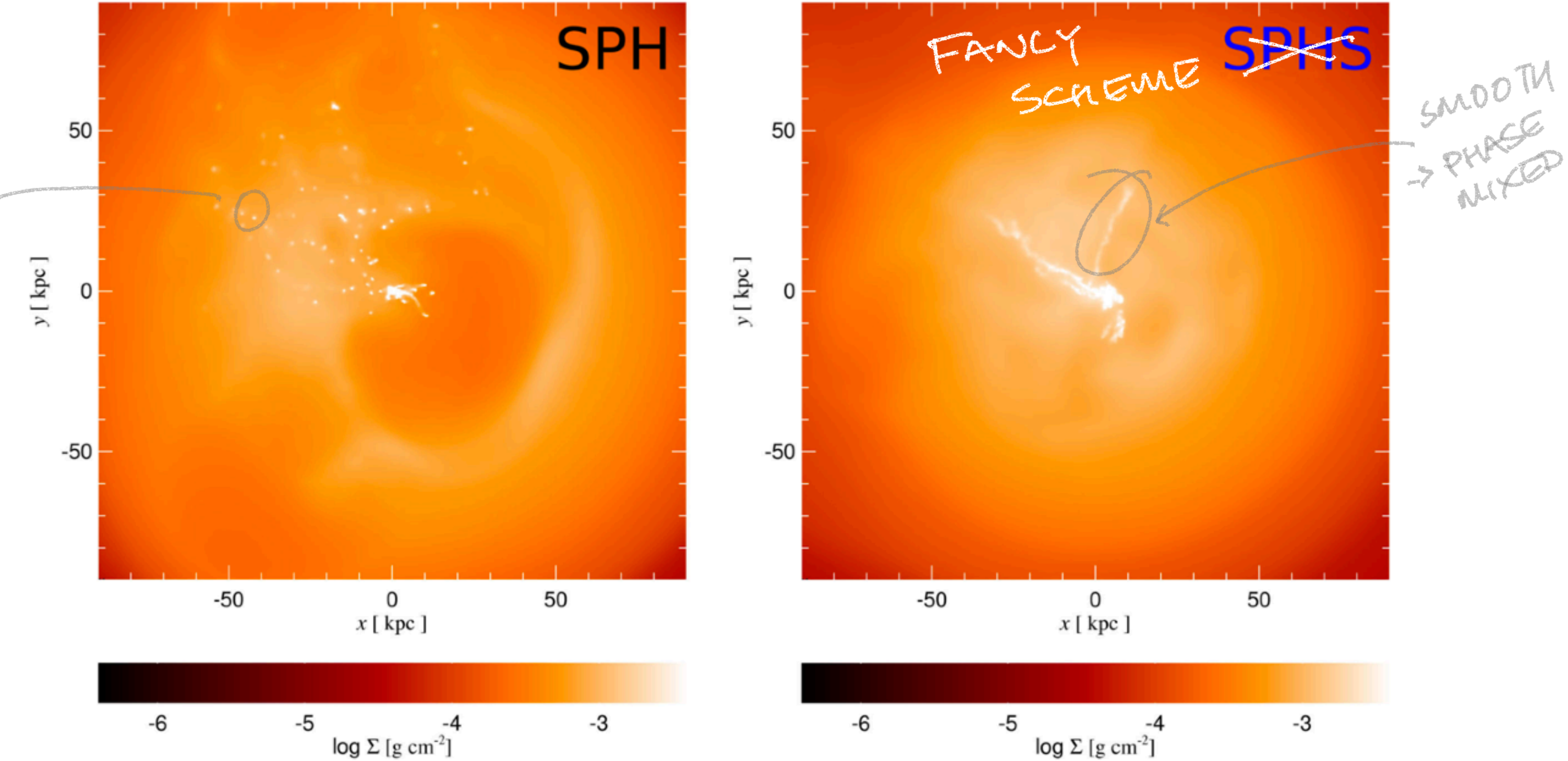
Density-Entropy (Gadget-2)

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left[\frac{f_i P_i}{\rho_i^2} \nabla_x W(\mathbf{x}_{ij}, h_i) + \frac{f_j P_j}{\rho_j^2} \nabla_x W(\mathbf{x}_{ij}, h_j) \right]$$

Pressure-Energy

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_j (\gamma - 1)^2 m_j u_j u_i \left[\frac{f_{ij}}{\bar{P}_i} \nabla_i W_{ij}(h_i) + \frac{f_{ji}}{\bar{P}_j} \nabla_i W_{ji}(h_j) \right]$$

What’s the Problem? (Physics)



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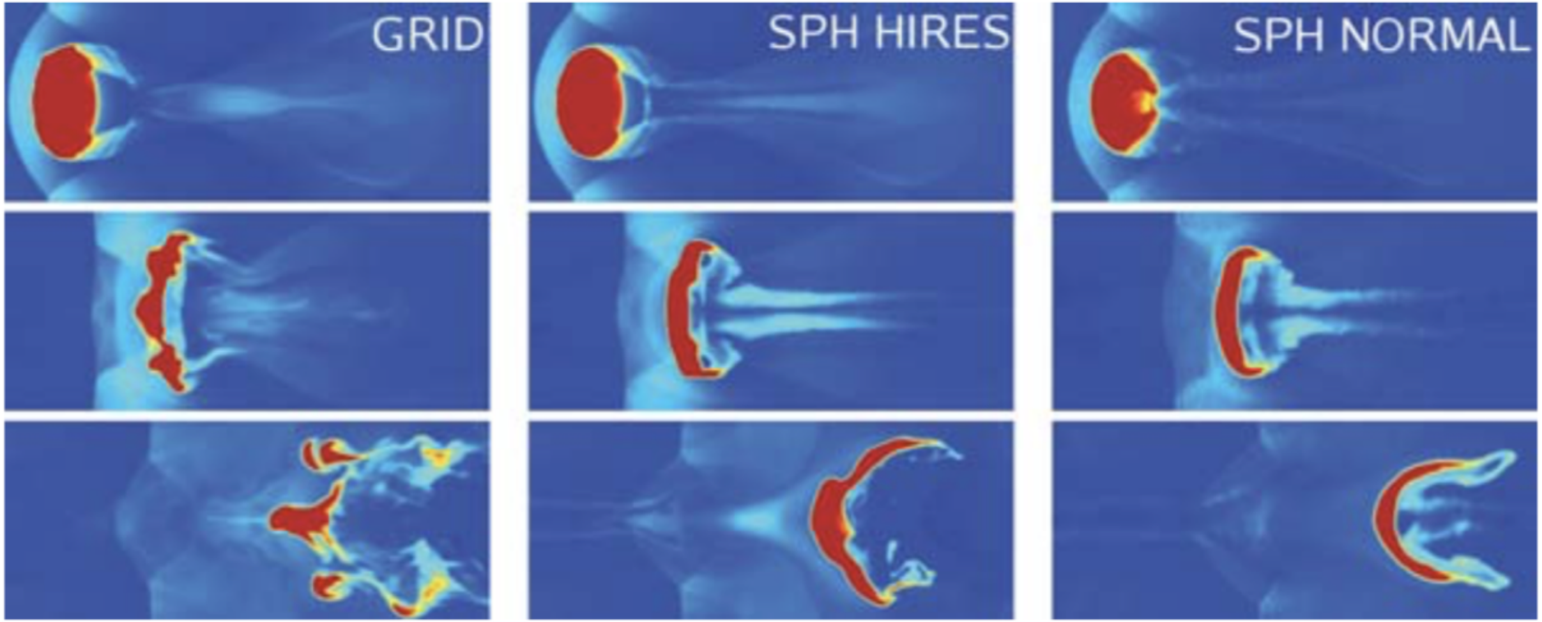
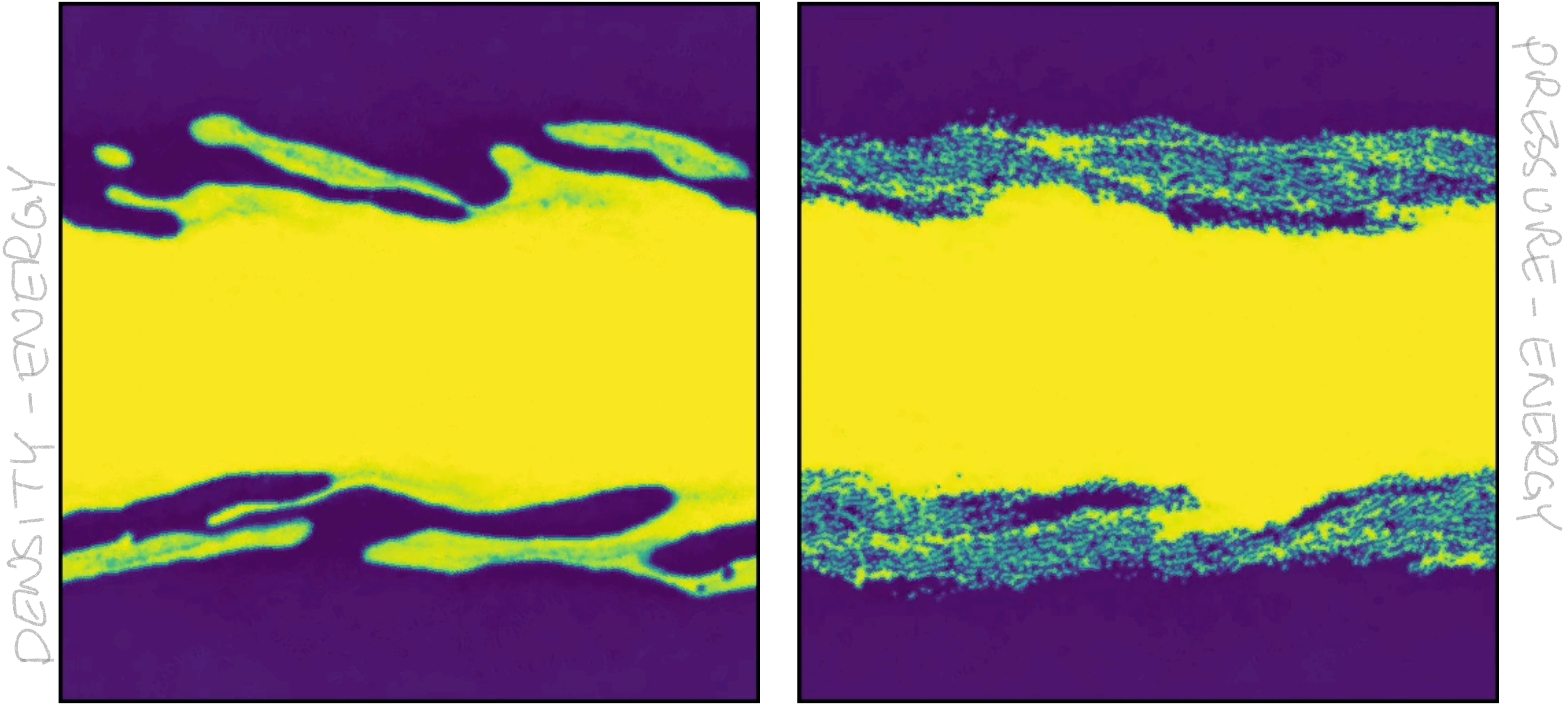
The Problem Visualised

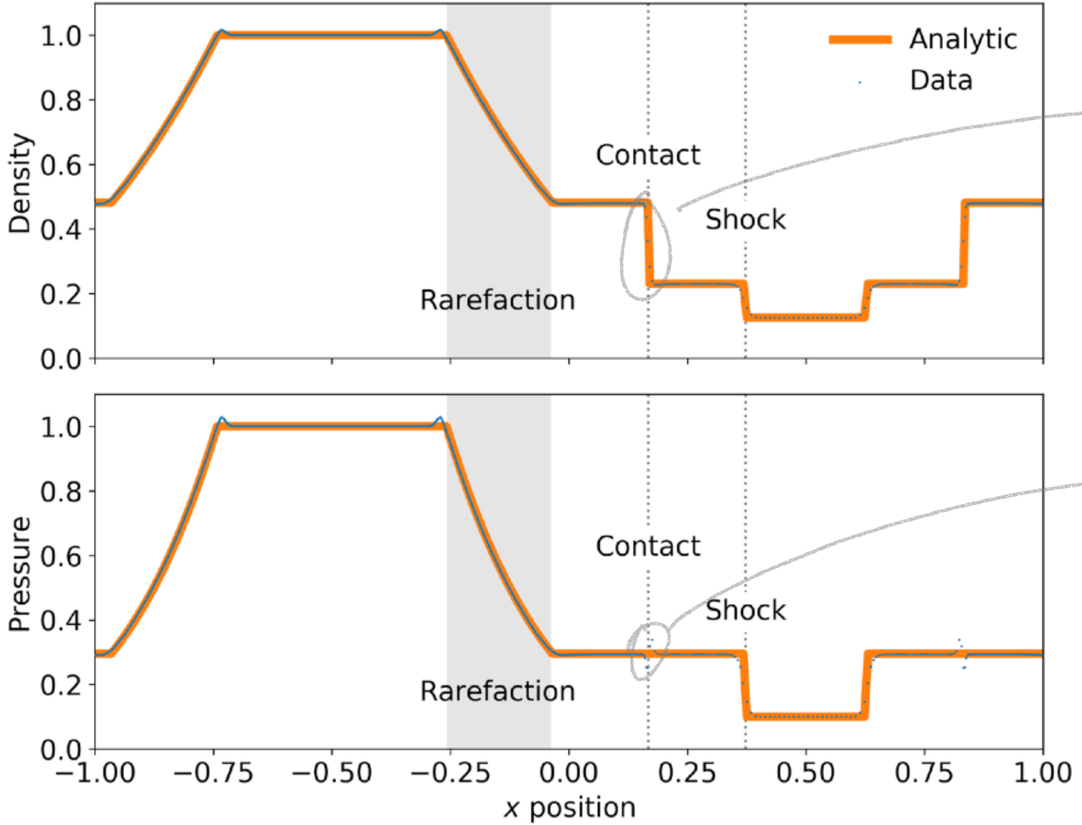


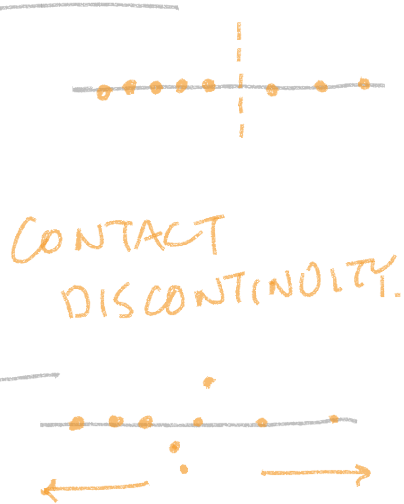
Fundamental differences between SPH and grid methods, Agertz+, 2007

One Level of Abstraction Down



Another Level Down (Truly Testable)





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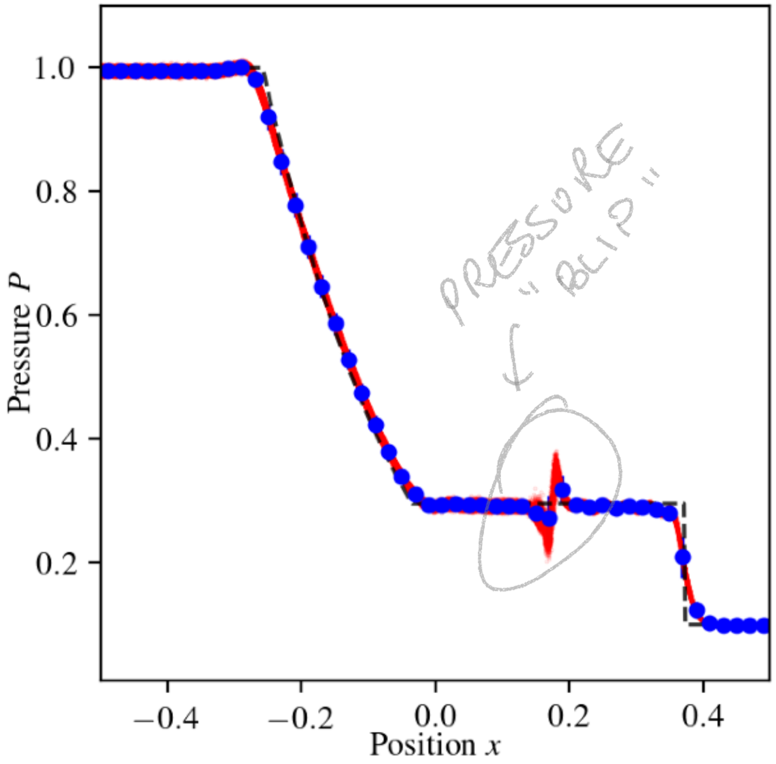
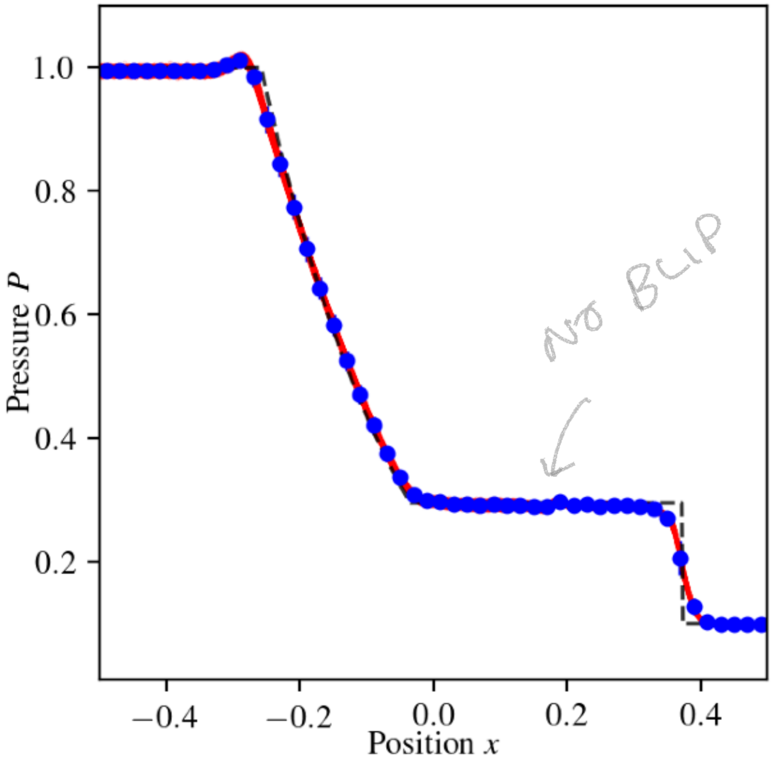




Why?

- Artificial surface tension, caused by the density (and hence pressure, from the EoS which is linear in density) being discontinuous
- We fix that by smoothing the pressure

Density-Entropy

$$\rho_i = \sum_j m_j W_{ij}$$

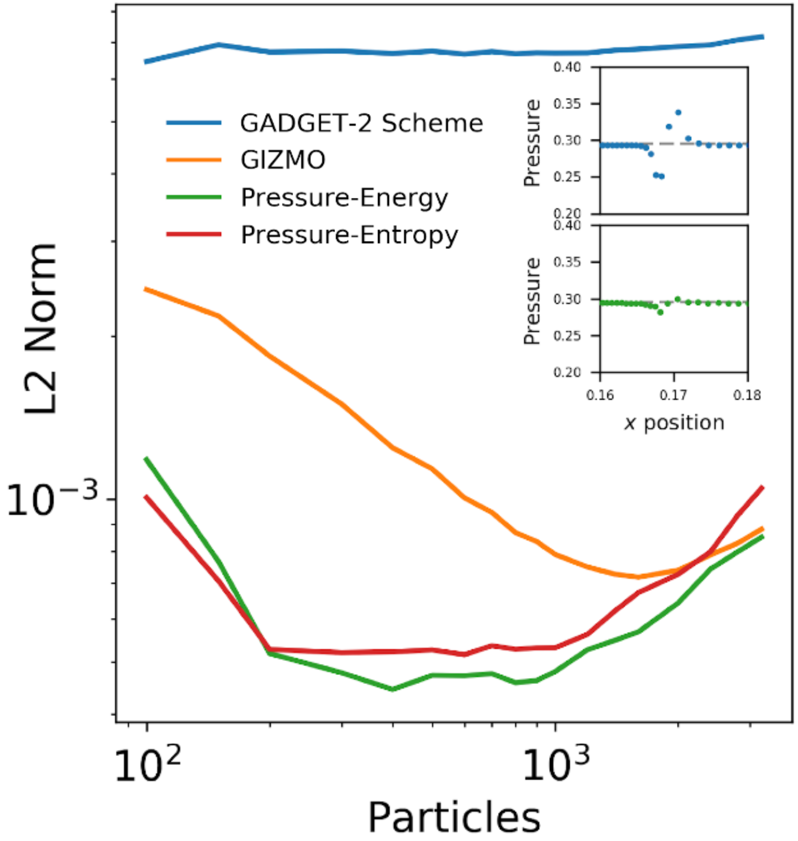
$$P_{\rm eos} = (\gamma - 1)\rho u$$

Pressure-Energy

$$\bar{P}_i = \sum_j m_j u_j (\gamma - 1) W_{ij}$$

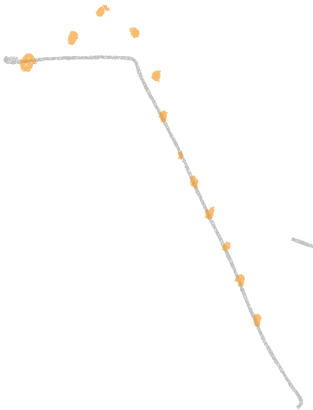
Does this Converge?

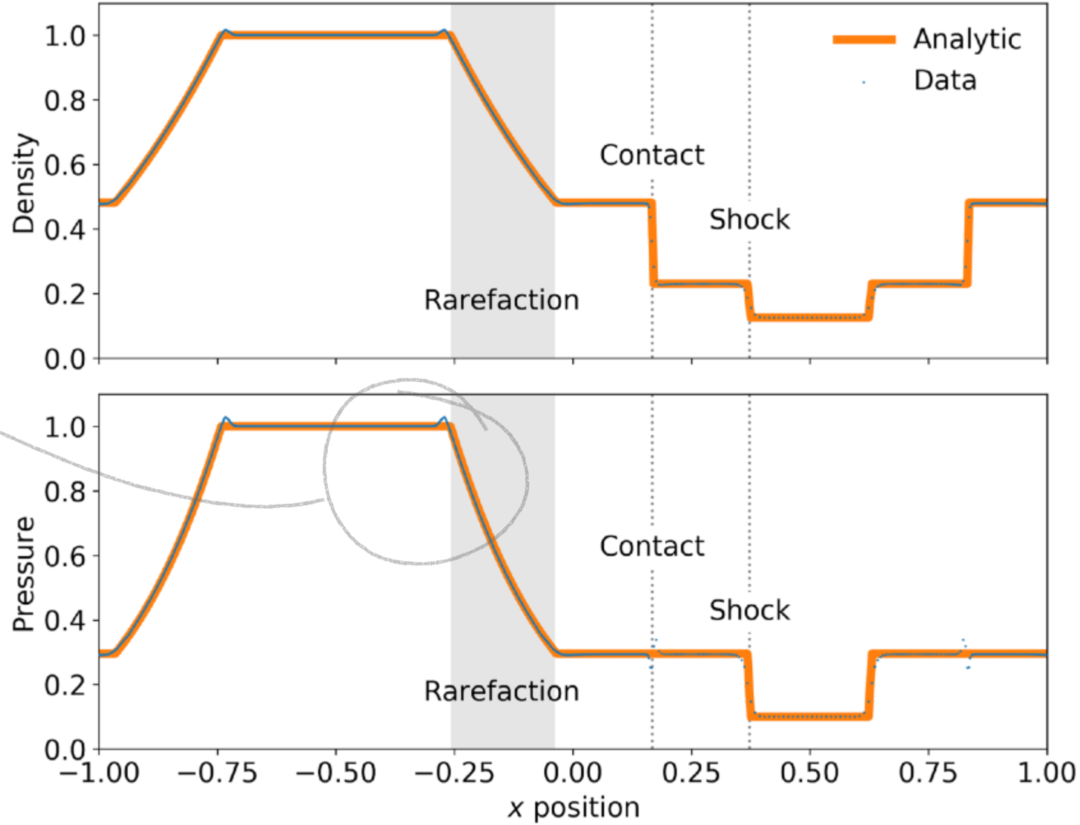
- Can look at this run with many different numbers of particles
- Note that the L2 norm *never* converges, stays constant



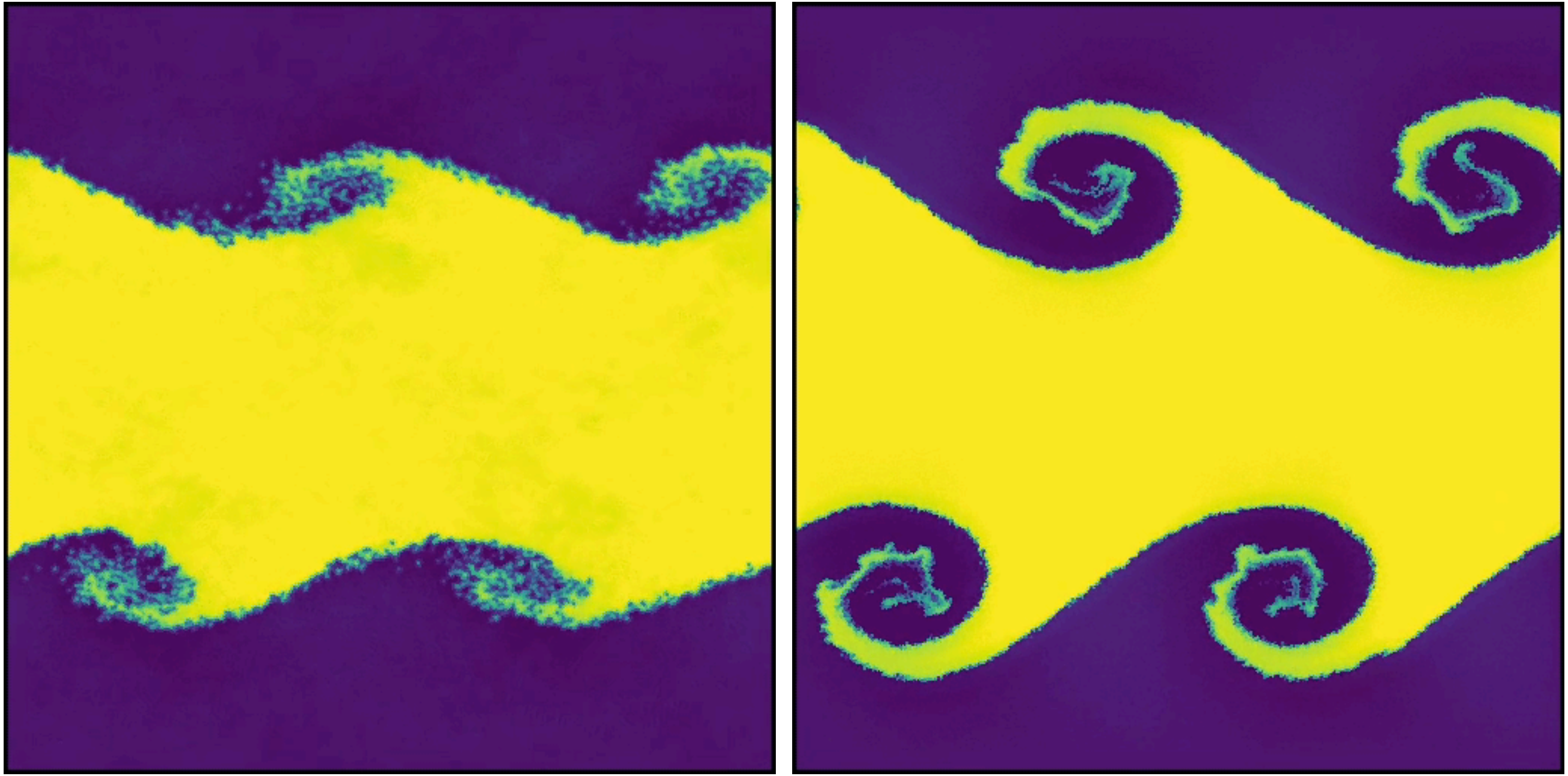
What About the Rest?

RAREFACTION WAVE



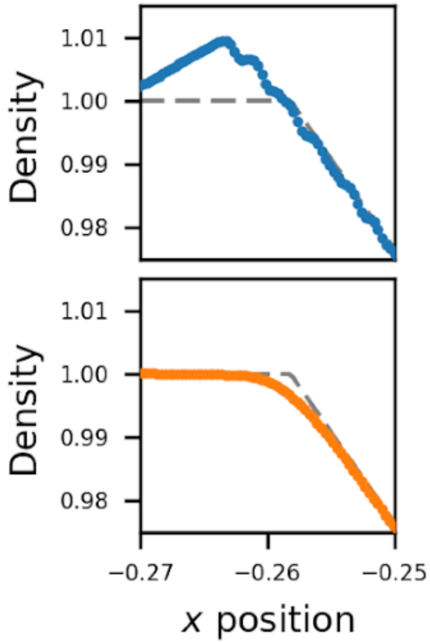


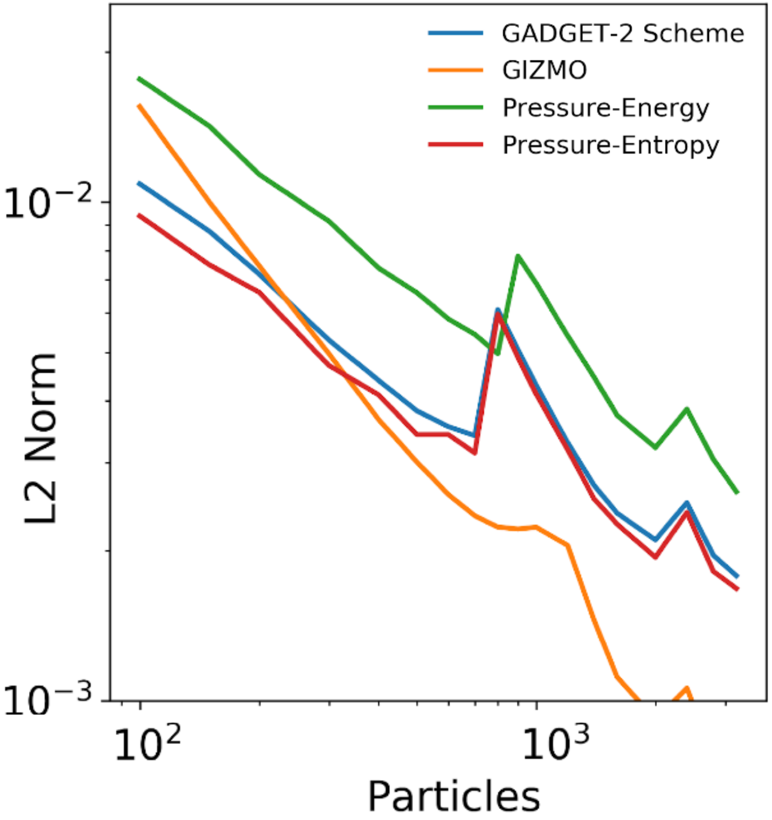
Can we do Better?



Can we do Better?

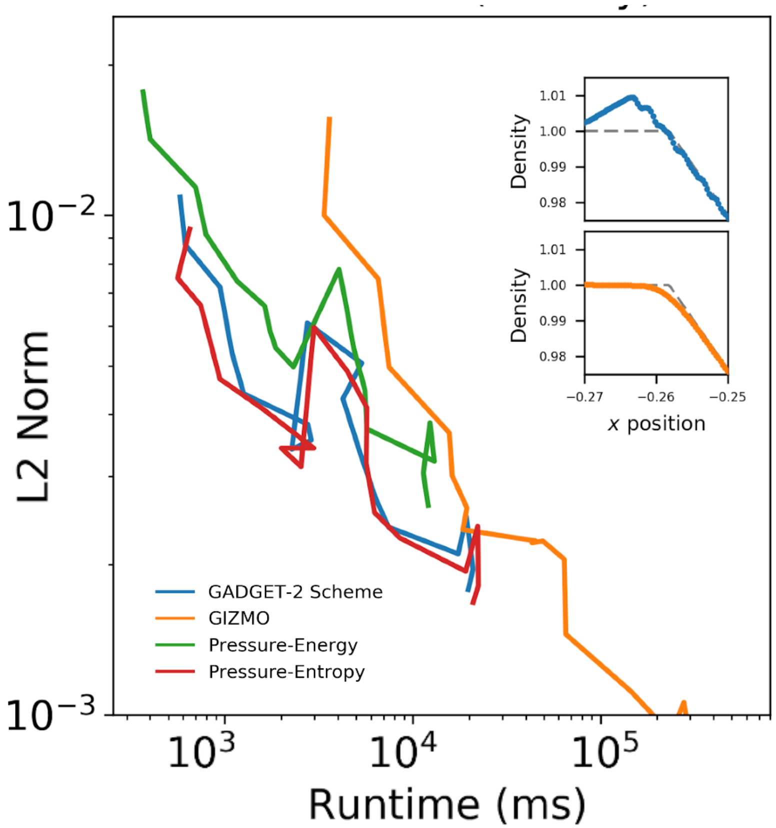
- Yes, of course!
- We can “do the hydrodynamics properly” (GIZMO, solves Riemann problem)





Is Particle Number the Correct Metric?

- Probably not!
- Best to compare at a fixed run-time, especially for more accurate schemes that are significantly more expensive
- At the moment, use SPH!



Conclusions

- SPH is widely used for astrophysical problems
- A minor change in the volume element chosen can fix a lot of issues (almost for free)
- We want to do this with many schemes, for many particles, in a reproducible way.